Decay Constants of B, B^* and D, D^* Mesons in Relativistic Mock Meson Model

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Abstract

We derive formulas for the decay constants f_P and f_V of pseudoscalar and vector mesons in the relativistic mock meson model. Using these formulas, we obtain f_P and f_V of B_s, B_d, D_s, and D_d mesons as functions of the mock meson parameter β. Then by using the values of β which are obtained by the variational method in the relativistic quark model, we obtain the decay constants f_P and f_V of the heavy mesons, and the corresponding ratios f_V/f_P. The results are compared with other calculations and existing experimental results.

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We study the decay constants of heavy pseudoscalar and vector mesons in the relativistic mock meson model of Godfrey, Isgur, and Capstick [1, 2, 3], in which the heavy meson state composed of a light quark q and a heavy antiquark $\bar{Q}$ is represented as

$$|M(\mathbf{K})>= \int d^3p\, \Phi(\mathbf{p})\, \chi_{s\bar{s}}\, \phi_{c\bar{c}}\, |q(\frac{m_q}{m}\mathbf{K}+\mathbf{p}, s)\, \bar{Q}(\frac{m_{\bar{Q}}}{m}\mathbf{K}-\mathbf{p}, \bar{s})>, \tag{1}$$

where $\mathbf{K}$ is the mock meson momentum, $m \equiv m_q + m_{\bar{Q}}$, and $\Phi(\mathbf{p})$, $\chi_{s\bar{s}}$, and $\phi_{c\bar{c}}$ are momentum, spin, and color wave functions respectively. We take the momentum wave function $\Phi(\mathbf{p})$ as a Gaussian wave function

$$\Phi(\mathbf{p}) = \frac{1}{(\sqrt{\pi}\beta)^{3/2}} e^{-\mathbf{p}^2/2\beta^2}. \tag{2}$$

The decay constant of the pseudoscalar and vector mesons, f_P and f_V respectively, are defined by

$$< 0|\, \bar{Q}\gamma^\mu\gamma_5 q\, |M_P(\mathbf{K}) >= f_P K^\mu, \quad < 0|\, \bar{Q}\gamma^\mu q\, |M_V(\mathbf{K}, \varepsilon) >= f_V m_V \varepsilon^\mu. \tag{3}$$

The meson state in (1) is written explicitly in the meson rest frame (where $\mathbf{p}_q = -\mathbf{p}_{\bar{Q}}$) as

$$\begin{aligned} |M_P(\mathbf{0}) > \;\; &= \;\; \sqrt{2m_P} \int \frac{d^3p_q}{(2\pi)^{3/2}\sqrt{2E_q\, 2E_{\bar{Q}}}}\; \Phi(\mathbf{p}_q)\; \frac{1}{\sqrt{N_c}} \\ &\times \frac{1}{\sqrt{2}}\, [a^\dagger_\uparrow(\mathbf{p}_q, c) b^\dagger_\downarrow(\mathbf{p}_{\bar{Q}}, \bar{c}) - a^\dagger_\downarrow(\mathbf{p}_q, c) b^\dagger_\uparrow(\mathbf{p}_{\bar{Q}}, \bar{c})]\, |0 >, \end{aligned} \tag{4}$$

where the arrow indicates a state with spin up (down) along a fixed axis and c is the colour index which is summed. Whereas we wrote the pseudoscalar meson state in (4), we can also write the vector meson state in the same way with the spin combinations for the vector states, which are given by $(\uparrow\uparrow)$, $1/\sqrt{2}\,(\uparrow\downarrow + \downarrow\uparrow)$ and $(\downarrow\downarrow)$. In (4) we adopted the normalization of the creation and annihilation operators given by $\{a(\mathbf{p}, s), a^\dagger(\mathbf{p}', s')\} = (2\pi)^3\, 2E\, \delta_{ss'} \delta^3(\mathbf{p}-\mathbf{p}')$, and then the meson state in (4) is normalized by $< M_P(\mathbf{0})|M_P(\mathbf{0}) >= 2m_P\, \delta^3(\mathbf{0})$, and also in the same way for the vector meson states.

Since we are concerned with the matrix elements in the left hand sides of (3) with the meson states in (4), it is convenient to represent the meson states by

$$\Psi_P \equiv - < 0|\, q\bar{Q}\, |M_P(\mathbf{0}) >, \quad \Psi_V \equiv - < 0|\, q\bar{Q}\, |M_V(\mathbf{0}) >, \tag{5}$$

with which the formulas in (3) are written as

$$Tr(\,\gamma^0\gamma_5\,\Psi_P\,) = f_P m_P, \quad Tr(\,\gamma^\mu\Psi_V\,) = f_V m_V \varepsilon^\mu. \tag{6}$$

If both two quarks inside the meson are static, the spinor combinations of $u(\mathbf{0})\bar{v}(\mathbf{0})$ for the pseudoscalar and vector meson states are given respectively as [4, 5]

$$P(\mathbf{0},\mathbf{0}) = -\frac{1}{\sqrt{2}}\frac{1+\gamma^0}{2}\gamma^5, \quad V(\mathbf{0},\mathbf{0},\varepsilon) = \frac{1}{\sqrt{2}}\frac{1+\gamma^0}{2}\;\not{\varepsilon}, \tag{7}$$

where the polarization vectors of the vector meson are given by $\varepsilon^\mu_\pm = (1/\sqrt{2})(0,1,\pm i,0)$ and $\varepsilon^\mu_3 = (0,0,0,1)$. However, since the quarks inside the mock meson represented by (4) are not static, we boost the spinors by using the formulas

$$u^{(\alpha)}(k) = \frac{\not{k}+m}{\sqrt{2m(m+E)}}u^{(\alpha)}(m,\mathbf{0}), \quad \bar{v}^{(\alpha)}(k) = \bar{v}^{(\alpha)}(m,\mathbf{0})\frac{-\not{k}+m}{\sqrt{2m(m+E)}}. \tag{8}$$

Then, through this procedure we obtain Ψ_P and Ψ_V in (5) as

$$\Psi_I = \sqrt{2m_I}\int \frac{d^3p_q}{(2\pi)^{3/2}}\,\Phi(\mathbf{p}_q)\frac{\sqrt{N_c}}{\sqrt{2E_q\,2E_{\bar{Q}}}}\,\frac{\not{p}_q+m_q}{\sqrt{2m_q(m_q+E_q)}}S_I\frac{-\not{p}_{\bar{Q}}+m_{\bar{Q}}}{\sqrt{2m_{\bar{Q}}(m_{\bar{Q}}+E_{\bar{Q}})}}, \tag{9}$$

where $I = P$ or V, and S_P and S_V are respectively $P(\mathbf{0},\mathbf{0})$ and $V(\mathbf{0},\mathbf{0},\varepsilon)$ in (7). By incorporating (9) into (6), we obtain the following formulas for the decay constants of pseudoscalar and vector mesons in the relativistic mock meson model:

$$f_I = \frac{2\sqrt{3}}{\sqrt{m_I}}\int \frac{d^3p}{(2\pi)^{3/2}}\,\Phi(\mathbf{p})\Big(\frac{E_q+m_q}{2E_q}\,\frac{E_{\bar{Q}}+m_{\bar{Q}}}{2E_{\bar{Q}}}\Big)^{1/2}\Big(1+a_I\,\frac{\mathbf{p}^2}{(E_q+m_q)(E_{\bar{Q}}+m_{\bar{Q}})}\Big), \tag{10}$$

where $I = P$ or V, and

$$a_P = -1, \quad a_V = +\frac{1}{3}. \tag{11}$$

We note that the formula for f_P in (10) and (11) was already obtained by Godfrey in Ref. [2], however we derived the formulas for both f_P and f_V systematically through the above procedure.

We can calculate f_P and f_V by using (10) and (11). When the meson and quark masses are given, f_P and f_V depend on the value of the parameter β in (2). We obtained numerically f_P and f_V of B_s, B_d, D_s, and D_d mesons as functions of β by using the meson [6] and the current quark [7] masses given by

$$\begin{aligned}
&m_{B_s} = 5.375\,\mathrm{GeV},\ m_{B_d} = 5.279\,\mathrm{GeV},\ m_{D_s} = 1.969\,\mathrm{GeV},\ m_{D_d} = 1.869\,\mathrm{GeV},\\
&m_{B_s^*} = 5.422\,\mathrm{GeV},\ m_{B_d^*} = 5.325\,\mathrm{GeV},\ m_{D_s^*} = 2.110\,\mathrm{GeV},\ m_{D_d^*} = 2.010\,\mathrm{GeV},\\
&m_b = 4.397\,\mathrm{GeV},\ m_c = 1.306\,\mathrm{GeV},\ m_s = 0.199\,\mathrm{GeV},\ m_d = 0.0099\,\mathrm{GeV}. \quad (12)
\end{aligned}$$

In Fig. 1 and 2 we present the results for the heavy mesons having s and d quarks as their light quark, repectively, and in Fig. 3 the ratios f_V/f_P.

In order to get the reliable results for the B and D meson decay constants in our calculation, it is necessary to know the accurate values of the parameter β. Capstick and Godfrey used the values of β obtained from various methods [1, 2, 3, 8] in this context. In this Letter we calculate the values of β by applying the variational method to the relativistic hamiltonian [9, 10]

$$H = \sqrt{\mathbf{p}^2 + {m_q}^2} + \sqrt{\mathbf{p}^2 + {m_{\bar{Q}}}^2} + V(r), \qquad (13)$$

where $\mathbf{r}$ and $\mathbf{p}$ are the relative coordinate and its conjugate momentum. The hamiltonian in (13) represents the energy of the meson in the meson rest frame, since in this reference frame the momentum of each quark is the same in magnitude as that of the conjugate momentum of the relative coordinate. In this variational calculation for β, we took the potential $V(r)$ in (13) from six different potential models, and the results of β have been presented in Ref. [10]. With these values of β and the meson and quark masses given in (12), except for the heavy quark masses (m_b and m_c) for which we used the values given in each potential model

as the fitted parameter values, we calculated the decay constants by using the formulas in (10) and (11). The results we obtained are as follows in MeV unit:

$$\begin{aligned} &f_{B_s} = 204 \pm 7,\ f_{B_d} = 173 \pm 7,\ f_{D_s} = 233 \pm 12,\ f_{D_d} = 191 \pm 12; \\ &f_{B_s^*} = 225 \pm 9,\ f_{B_d^*} = 194 \pm 8,\ f_{D_s^*} = 298 \pm 11,\ f_{D_d^*} = 262 \pm 10. \end{aligned} \tag{14}$$

We also present the detailed results in Table 1, from which we get the ratios of the pseudoscalar and vector meson decay constants:

$$\frac{f_{B_s^*}}{f_{B_s}} = 1.11\pm0.00,\ \frac{f_{B_d^*}}{f_{B_d}} = 1.12\pm0.01,\ \frac{f_{D_s^*}}{f_{D_s}} = 1.28\pm0.03,\ \frac{f_{D_d^*}}{f_{D_d}} = 1.37\pm0.04. \tag{15}$$

We see that the ratios for D mesons are bigger than those for B mesons, which can be understood by the fact that the second term in the last factor of (10) contributes more in the case of D mesons. (10) also gives the inequality

$$\sqrt{m_V} f_V \geq \sqrt{m_P} f_P, \tag{16}$$

in which the equality holds in the static limit, that is, when two quarks inside the meson are static. Isgur and Wise obtained the similar relation, $m_V f_V = m_P f_P$ (which is written in our convention of f_V), in the static limit through the heavy quark effective theory [11]. We compare our results in (15) with other calculations in Table 2: Neubert's calculation by the heavy quark effective theory [5], and the lattice results of the ELC group [12].

We also compare our results by summarizing the results of the pseudoscalar meson decay constants from various different calculations in Table 3. The second row is the results of the original calculations of Capstick and Godfrey [3], which were obtained by the same method as ours, but with the different values of the parameter β. In this Letter, by obtaining the formulas for f_P and f_V in (10) and (11) through the formalism of (6) and (9), we could calculate both pseudoscalar and vector meson decay constants. Dominguez did the calculation using the QCD sum rules [13]. The fourth and fifth rows are from the lattice calculations of the

UKQCD [14] and the BLS [15] groups. In the last two rows we wrote the experimental results of the WA75 [16] and the CLEO [17] groups. We find that our result of f_{D_s} agrees very well with the WA75 result. Finally we mention that we get the value of the double ratio $(f_{B_s}/f_{B_d})\,/\,(f_{D_s}/f_{D_d})$ as 0.967 from our results in Table 3, which is exactly the same as the value that Grinstein obtained by the heavy quark effective theory [18].

Acknowledgements

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Model	f_{B_s}	$f_{B_s^*}$	f_{B_d}	$f_{B_d^*}$	f_{D_s}	$f_{D_s^*}$	f_{D_d}	$f_{D_d^*}$
A (Eich.)[19]	198	218	170	187	242	300	200	264
B (Hagi.)[20]	190	209	160	180	210	276	170	243
C (Power 1)[21]	208	229	177	199	249	313	207	276
D (Power 2)[22]	213	237	181	204	234	301	190	262
E (Log.)[23]	206	229	175	198	229	296	187	259
F (Rich.)[24]	206	228	175	197	231	300	190	265
(Average)	204±7	225±9	173±7	194±8	233±12	298±11	191±12	262±10

Table 1: The pseudoscalar and vector meson decay constants (MeV) in six different potential models.

	$f_{B_s^*}/f_{B_s}$	$f_{B_d^*}/f_{B_d}$	$f_{D_s^*}/f_{D_s}$	$f_{D_d^*}/f_{D_d}$
This Work	1.11±0.00	1.12±0.01	1.28±0.03	1.37±0.04
Neubert [5]	—	1.07±0.02	—	1.35±0.05
ELC [12]	—	1.12±0.05	—	1.30±0.06

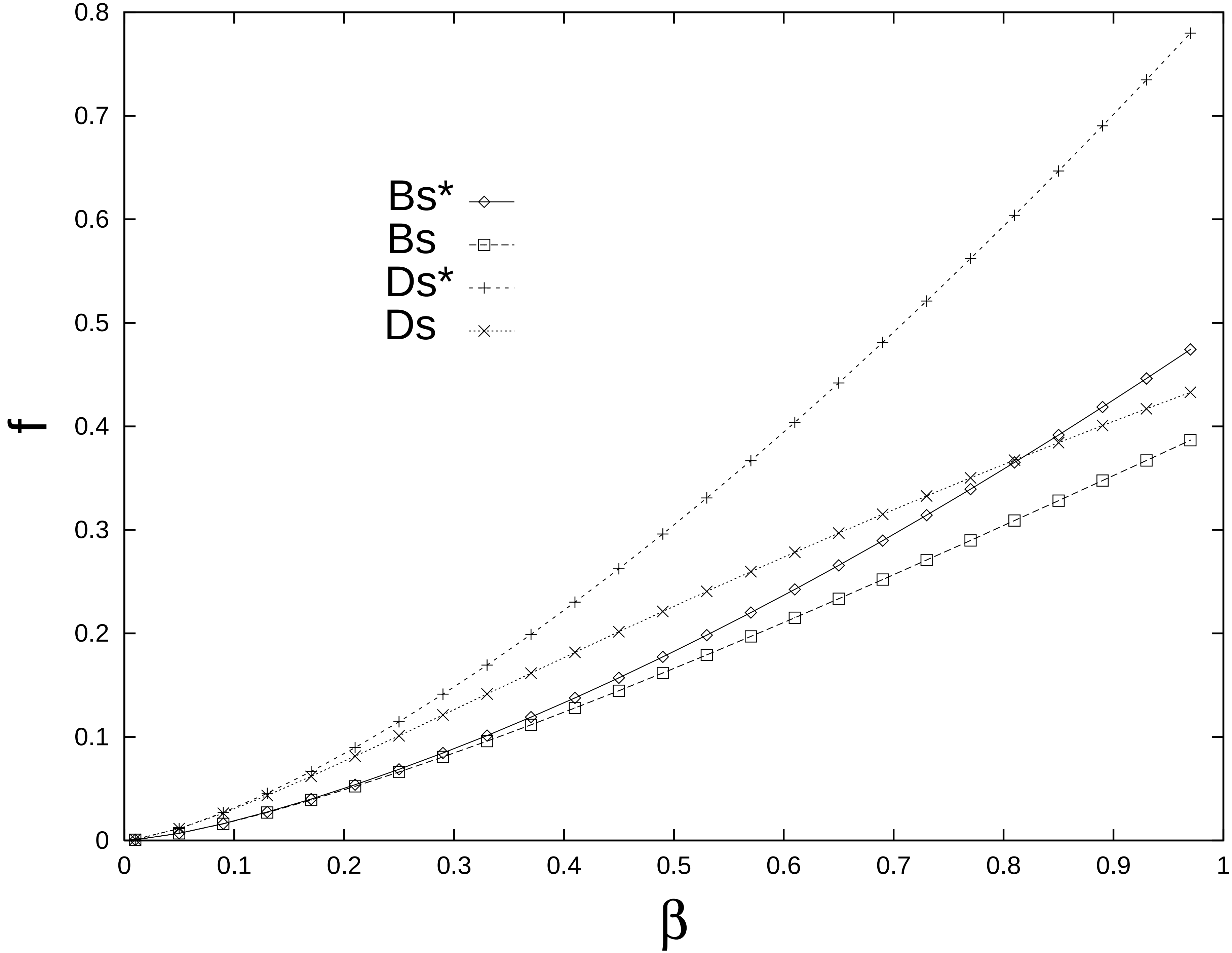
Table 2: The ratios of the decay constants of vector and pseudoscalar mesons.

	f_{B_s}	f_{B_d}	f_{B_s}/f_{B_d}	f_{D_s}	f_{D_d}	f_{D_s}/f_{D_d}
This Work	204±7	173±7	1.18±.01	233±12	191±12	1.22±.01
Cap. Godf. [3]	210±20	155±15	1.35±.18	290±20	240±20	1.21±.13
Doming. [13]	193±28	158±25	1.22±.02	222±48	187±48	1.21±.06
UKQCD[14]	$194\,^{+6+62}_{-5-9}$	$160\,^{+6+53}_{-6-19}$	$1.22\,^{+.04}_{-.03}$	$212\,^{+4+46}_{-4-7}$	$185\,^{+4+42}_{-3-7}$	1.18±.02
BLS [15]	207±9 ± 40	187±10 ± 37	1.11±.02 ± .05	230±7 ± 35	208±9 ± 37	1.11±.02 ± .05
WA75 [16]	—	—	—	232±45 ± 52	—	—
CLEO [17]	—	—	—	344±37 ± 67	—	—

Table 3: The values (MeV) and ratios of the decay constants from different calculations and experimental results.

Fig. 2. $f_{B_d^*}$, f_{B_d}, $f_{D_d^*}$, and f_{D_d} as functions of the parameter β.

Fig. 3. $f_{B_s^*}/f_{B_s}$, $f_{B_d^*}/f_{B_d}$, $f_{D_s^*}/f_{D_s}$, and $f_{D_d^*}/f_{D_d}$ as functions of the parameter β.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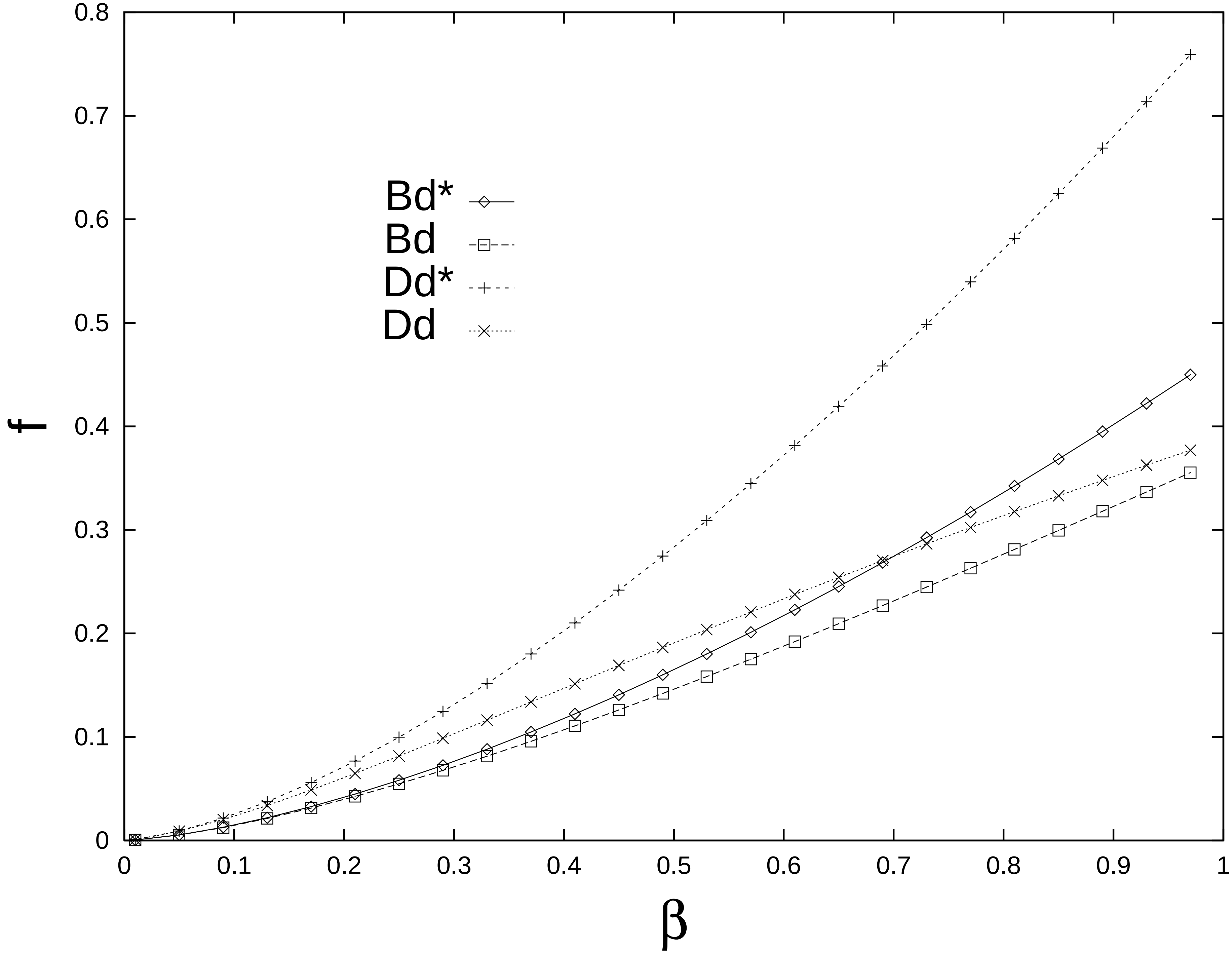

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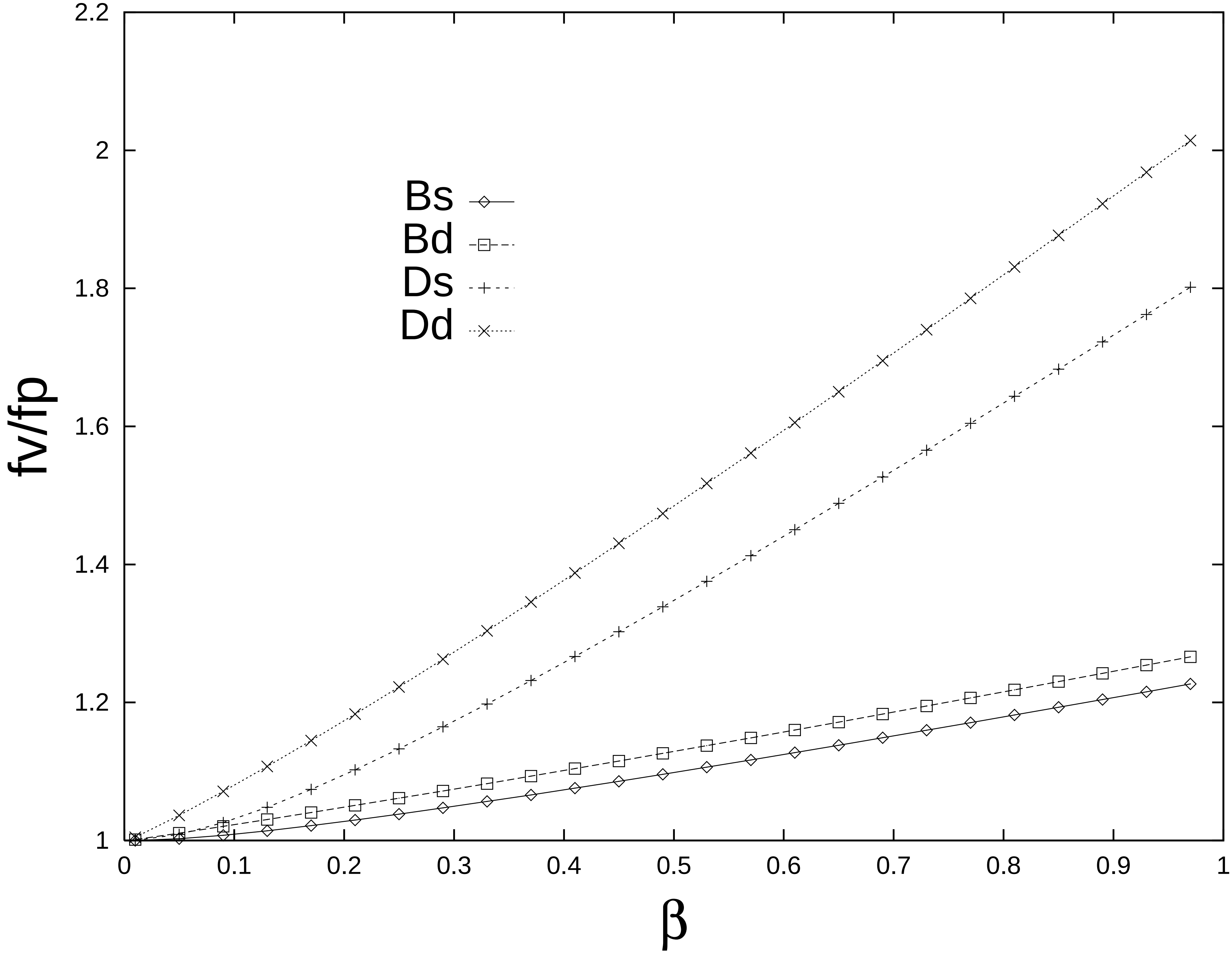

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